

Pseudo-Goldstones from Supersymmetric Wilson Lines on 5D Orbifolds

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Abstract

We consider a $U(1)$ gauge theory on the five dimensional orbifold $\mathcal{M}_4 \times S^1/Z_2$, where A_5 has even Z_2 parity. This leads to a light pseudoscalar degree of freedom $W(x)$ in the effective 4D theory below the compactification scale arising from a gauge-invariant brane-to-brane Wilson line. As noted by Arkani-Hamed *et al.* in the non-supersymmetric S^1 case the 5D bulk gauge-invariance of the underlying theory together with the non-local nature of the Wilson line field leads to the protection of the 4D theory of $W(x)$ from possible large global-symmetry violating quantum gravitational effects. We study the S^1/Z_2 theory in detail, in particular developing the supersymmetric generalization of this construction, involving a pseudoscalar Goldstone field (the ‘axion’) and its scalar and fermion superpartners (‘saxion’ and ‘axino’). The global nature of $W(x)$ implies the absence of independent Kaluza-Klein excitations of its component fields. The non-derivative interactions of the (supersymmetric) Wilson line in the effective 4D theory arising from $U(1)$ charged 5D fields Φ propagating between the boundary branes are studied. We show that, similar to the non-supersymmetric S^1 case, these interactions are suppressed by $\exp(-\pi R m_\Phi)$ where πR is the size of the extra dimension.

1 Introduction

The study of both non-supersymmetric and supersymmetric gauge field theories on the five dimensional manifold $\mathcal{M}_4 \times S^1$ and orbifolds $\mathcal{M}_4 \times S^1/Z_2$ and $\mathcal{M}_4 \times S^1/(Z_2 \times Z'_2)$ has attracted much recent interest. One application of gauge theories on $\mathcal{M}_4 \times S^1/(Z_2 \times Z'_2)$ has been to provide novel solutions to the problems of standard 4D supersymmetric grand-unified (GUT) theories [1, 2, 3, 4]. The GUT gauge symmetry is now realized in 5 or more space-time dimensions and broken to the Standard Model (SM) gauge group by compactification on an orbifold, utilizing boundary conditions that violate the GUT-symmetry. In the most studied case of 5 dimensions both the GUT group and 5D supersymmetry are broken by compactification on $S^1/(Z_2 \times Z'_2)$, leading to a 4D N=1 SUSY model with SM gauge group. This construction provides elegant solutions to the problems of conventional GUTs with Higgs breaking, including doublet-triplet splitting, dimension-5 proton decay, and Yukawa unification in the first two generations, while maintaining, at least at leading order, the desired gauge coupling unification [2, 5]. The hierarchy between the strong coupling scale M of the 5D gauge theory and the compactification scale $1/R$ ($MR \sim 10^2 \cdots 10^3$) can also be used to generate a fermion mass hierarchy [6].

In another line of development, Arkani-Hamed *et al.* [7] studied a 5D $U(1)$ gauge theory compactified on $\mathcal{M}_4 \times S^1$ and the gauge-invariant Wilson line phase Θ associated to the zero mode of A_5

$$\exp(-i\Theta) \equiv \exp\left(-i \int dy A_5\right). \quad (1)$$

The phase Θ becomes a field in the 4D effective theory when the A_5 zero-mode is allowed to have a slow variation as a function of the coordinates on \mathcal{M}_4 . The associated 4D field theory has the interesting property that it inherits a global shift symmetry $\Theta \rightarrow \Theta + c$ from the 5D underlying gauge symmetry, and this leads to the phenomenologically interesting possibility that the global shift symmetry is protected from dangerous local 5D quantum-gravitational symmetry violating effects [8] by the underlying 5D gauge symmetry. In particular, Arkani-Hamed *et al.* [7] utilized this construction to build interesting new models of inflation that at least in part address the problems of traditional 4D inflationary model building. Later papers [9, 10] generalized this to supersymmetric models of inflation on $\mathcal{M}_4 \times S^1$ and to applications to quintessence and axion model building [11, 12]. In our present work we wish to further generalize these ideas to the supersymmetric case on Z_2 orbifolds of $\mathcal{M}_4 \times S^1$, bringing the models closer to the orbifold GUT constructions.

In the orbifold case the most commonly studied situation is where the orbifold parity operation is chosen such that the A_5 component of the 5D gauge field is odd, implying the absence of a A_5 zero mode (the A_μ component is then even, thus leading to an A_μ zero mode and thus a 4D gauge theory). However as argued in Ref.[12] in the context of 5D theories of axions the other choice, where the 4D gauge zero mode A_μ is projected out by taking A_5 to be *even* is also phenomenologically interesting. When A_5 has even Z_2 parity, there exists a zero mode $A_5(x, y) = A_5(x, 0)$ in the spectrum of the theory. In this paper, we consider a $U(1)$ gauge field on the five dimensional orbifold $\mathcal{M}_4 \times S^1/Z_2$,

where we choose A_5 to have even Z_2 parity, leading to a light physical gauge-invariant Wilson line degree of freedom in the effective 4D theory below the compactification scale.¹ Similar to the S^1 case [7] the 5D bulk gauge-invariance of the underlying theory together with the non-local nature of the Wilson line field leads to the protection of the 4D theory of the Wilson line field from possible global-symmetry violating quantum gravitational effects, at least if the size of the 5th dimension, πR is large compared to the fundamental 5D Planck scale M_5 . In the following sections, we study the S^1/Z_2 theory in detail, in particular developing the supersymmetric generalization of this construction, and we explore the effect of the Wilson line on the effective 4D theory in analogy to the case of S^1 [7].

2 Non-supersymmetric Wilson lines on S^1/Z_2

We want to study Wilson lines arising from the fifth component, A_5 , of a $U(1)$ 5D gauge field A_M and its coupling to bulk bosons and fermions Φ , Ψ and brane bosons and fermions ϕ , ψ . (Our conventions are given in Appendix A.) In order to have a zero-mode, the A_5 component must have even Z_2 parity $A_5(-y) = A_5(y)$ which fixes the parities of the gauge parameter Λ and the A_μ components to be odd due to the gauge transformation $A_M \rightarrow A_M + \partial_M \Lambda$:

$$\Lambda(-y) = -\Lambda(y) \quad A_\mu(-y) = -A_\mu(y). \quad (2)$$

Thus the gauge transformation is forced to be trivial at the S^1/Z_2 orbifold fixed points, $\Lambda(0) = 0$ and $\Lambda(\pi R) = 0$, and this is the crucial fact that will allow us to define gauge-invariant Wilson *lines* that appear in the 4D effective theory as additional fields.

Considering scalar and fermion charged bulk fields Φ and Ψ we are free to choose their Z_2 parities defined by $\Phi(-y) = \pm\Phi(y)$ for bosons and

$$\Psi(-y) = \pm i\gamma_5 \Psi(y) \quad (3)$$

for fermions. Using the fact that $i\gamma_5 \Psi_{L,R} = \pm \Psi_{L,R}$, it is clear that even parity in Eq.(3) corresponds to choosing Ψ_L even and Ψ_R odd and vice versa for odd parity. Consistency with Eq.(2) demands that the associated gauge transformations and covariant derivatives for Φ and Ψ are given by:

$$\Phi \rightarrow \exp(-i\epsilon(y)q\Lambda)\Phi \quad \Psi \rightarrow \exp(-i\epsilon(y)q\Lambda)\Psi \quad (4)$$

and $D_M = \partial_M + iq\epsilon(y)A_M$, where q is the $U(1)$ charge of the respective bulk field, and the step function $\epsilon(y)$ is

$$\epsilon(y) = \begin{cases} -1 & \text{for } -\pi R < y < 0 \\ 0 & \text{for } y = 0 \\ +1 & \text{for } 0 < y < \pi R \end{cases}$$

¹Our treatment trivially extends to a compactification on $S^1/(Z_2 \times Z'_2)$ with A_5 even at both fixed points.

In summary, the parities for the bulk fields are:

$$\begin{array}{c|c|c|c|c|c} A_\mu & A_5 & \Lambda & \Phi & \Psi_L & \Psi_R \\ \hline - & + & - & \text{free} & +/- & -/+ \end{array} \quad (5)$$

Because $\Lambda(0) = 0 = \Lambda(\pi R)$ any brane-localized fields on the orbifold branes at $y = 0, \pi R$ are $U(1)$ gauge singlets.

The Wilson line of a gauge theory can be defined as the parallel transport of the gauge connection along a path C parameterized by $s \in [0, 1]$ from $x(0) = x_i$ to $x(1) = x_f$ and thus as a solution of

$$\frac{dx^M}{ds} D_M W(C) = 0, \quad (6)$$

where D_M is the covariant derivative. According to this definition, the Wilson loop transforms as

$$W(C) \rightarrow U(\Lambda(x_f)) W(C) U^{-1}(\Lambda(x_i)). \quad (7)$$

for a gauge transformation Λ . For a covariant derivative defined as $D_M = \partial_M + iA_M$, the solution to Eq.(6) is found to be

$$W(C) \equiv \exp \left(-i \int_C ds \dot{x}^M A_M \right). \quad (8)$$

A gauge invariant operator can be defined from Eq.(8) on a general manifold by choosing the path C to be closed, yielding the well known Wilson loop. However on an orbifold with a gauge parameter Λ which vanishes at (certain) singularities an open Wilson line linking those singularities is gauge invariant despite the path C not being closed. In our S^1/Z_2 case we can define a gauge-invariant Wilson line phase $\Theta(x)$ by

$$e^{-i\Theta(x)} \equiv \exp \left(-i \int_0^{\pi R} A_5(x, y) dy \right). \quad (9)$$

As in the $M_4 \times S^1$ case [7], the Wilson line phase $\Theta(x)$ implies the existence of a scalar field of the 4D effective theory. (Here we implicitly make the assumption that the variation in x is slow compared to size πR of the S^1/Z_2 orbifold direction, so that a 4D effective field theory description is valid.) Note that $\Theta(x)$ is a *non-local* field from the 5D perspective. By calculating the propagators of charged bulk fields we show below how the Wilson line phase can enter the 4D effective Lagrangian starting from local 5D interactions.

The kinetic and interaction terms for Θ in the 4D effective action follow from the 5D action, which has the form

$$S = \int d^4x \int_0^{\pi R} dy \left[\mathcal{L}_{5D} + \delta(y M_5) \mathcal{L}_0 + \delta([y - \pi R] M_5) \mathcal{L}_{\pi R} \right] \quad (10)$$

where the brane terms $\mathcal{L}_{0, \pi R}$ depend only on Z_2 even bulk fields and brane localized fields, and M_5 is the cutoff scale of the 5D theory. The bulk action contains the gauge term

$$S_{5D, \text{gauge}} = -\frac{1}{4g^2} \int d^4x dy F_{MN} F^{MN} + \dots \supset -\frac{2}{4g^2} \int d^4x dy F_{\mu 5} F^{\mu 5}. \quad (11)$$

To identify the 4D kinetic term for Θ following from this action it is useful to choose a ‘uniform gauge’ (constant in y) representative of the gauge configuration A_5 with specified $\Theta(x)$

$$A_5(x, y) \equiv A_5(x) = \frac{\Theta(x)}{\pi R}. \quad (12)$$

Then Eq.(11) implies a kinetic term for Θ :

$$\int d^4x \left(-\frac{1}{2\pi R g^2} (\partial_\mu \Theta)^2 \right) = \int d^4x \left(-\frac{1}{2} (\partial_\mu \sigma)^2 \right). \quad (13)$$

where we have canonically normalized the 4D field by defining $\sigma(x) \equiv f\Theta(x)$ with

$$f^2 = \frac{1}{\pi R g^2}. \quad (14)$$

Because of the gauge invariance of the original 5D action, and the non-local nature of $\Theta(x)$, potential terms for the Wilson line phase, $\sigma(x)$, in the 4D action only arise from the propagation of charged bulk fields from one brane to the other. Thus we need to evaluate the propagator for charged bulk fields. All our arguments can be illustrated using scalar fields and so the relevant matter terms in the 5D bulk Lagrangian are

$$\mathcal{L}_{5D, \text{matter}} = -(D_M \Phi)^\dagger (D^M \Phi) - m_\Phi^2 \Phi^\dagger \Phi. \quad (15)$$

The propagator is the Greens function for the equation of motion (with as usual a suitable Feynman $i\epsilon$ prescription imposed which we leave implicit). We only need the propagator from the $y = 0$ brane to the $y = \pi R$ brane, but it is convenient to consider propagation from the $y = 0$ brane to an arbitrary position y in the bulk. Fourier transforming to momentum space p^μ in the x^μ directions, but leaving the 5th direction in coordinate space, the Greens function, $\Delta_\Phi(p, y)$, for a scalar in the slowly varying A_5 background solves

$$[\partial_5^2 - p^2 + 2iq\epsilon(y)A_5\partial_5 - q^2A_5^2 - m_\Phi^2] \Delta_\Phi(y) = -\delta(y), \quad (16)$$

where we are here working in uniform gauge with $A_5(x, y) = A_5(x)$. For the relevant case of even parity fields, Neumann boundary conditions $\partial_5 \Delta_\Phi(y) = 0$ have to be imposed at $y = 0, \pi R$. The solution specialized to the case of interest, $y = \pi R$, is

$$\Delta_{\Phi,+}(p; 0, \pi R) = \frac{p' \exp(-iqA_5\pi R)}{(p'^2 + q^2A_5^2) \sinh(\pi R p')}. \quad (17)$$

where p' is given by $p'^2 = p^2 + m_\Phi^2$.

The most important limit of the expression Eq.(17) is when the separation between the two branes is large compared to the inverse mass of the bulk particle, $\pi R m_\Phi \gg 1$:

$$\Delta_{\Phi,+}(p; 0, \pi R) = \frac{2e^{-\pi R m_\Phi}}{m_\Phi \left(1 + \frac{q^2 g^2 \sigma(x)^2}{\pi R m_\Phi^2} \right)} \exp(-iq\sqrt{g^2 \pi R} \sigma(x)) + \mathcal{O}(p^2), \quad (18)$$

where we have switched to the canonically normalized 4D field $\sigma(x)$. For values of σ where the phase is $\mathcal{O}(1)$ or less the condition $\pi R m_\Phi \gg 1$ implies the further simplification

$$\Delta_{\Phi,+}(p; 0, \pi R) = \frac{2e^{-\pi R m_\Phi}}{m_\Phi} \exp(-iq\sqrt{g^2 \pi R} \sigma(x)) + \mathcal{O}(p^2). \quad (19)$$

Thus as claimed the propagation of charged bulk fields with even parity from brane to brane leads to terms involving the gauge-invariant Wilson-line phase

$$\exp(-iq\sqrt{g^2\pi R}\sigma(x)). \quad (20)$$

As expected, in the limit that $\pi R \gg 1/m_\Phi$, the overall amplitude of the Greens function is suppressed by $\exp(-m_\Phi\pi R)$.

Contributions to the 4D effective field theory involving the Wilson line phase arise when the charged massive even-parity bulk fields which we integrate out couple to brane-localized operators j and j' at $y = 0$ and $y = \pi R$:

$$S_{int} = \int d^4x \int_0^{\pi R} dy \left[\lambda j(x) \Phi(x, y) \delta(y M_5) + \lambda' j'(x) \Phi(x, y) \delta((y - \pi R) M_5) + \text{h.c.} \right] \quad (21)$$

A tree level Feynman graph in the 5D theory coupling the charged bulk scalar to currents on the branes then results to leading order and for $m_\Phi\pi R \gg 1$ in an interaction in the 4D effective theory given by

$$\lambda\lambda'^\dagger \frac{e^{-m_\Phi\pi R}}{m_\Phi} \int d^4x j(x) e^{-iq\sqrt{g^2\pi R}\sigma(x)} j'^\dagger(x) + \text{h.c.} \quad (22)$$

The most important aspect of this non-derivative interaction of σ is that it is non-local from the 5D perspective, requiring the propagation of charged fields across the 5D bulk. This ensures that these interaction terms for σ are protected from large quantum gravity corrections, as these too must be non-local in 5D, and are thus suppressed by the action of a black hole or wormhole or other 5D quantum gravitational effect of linear size $\pi R \gg 1/M_5$. Second, the size of the non-derivative interactions generated by charged bulk fields is exponentially suppressed by the mass m_Φ for moderately large bulk, so effects that lift the masslessness of $\sigma(x)$ can be naturally exponentially suppressed. Third, a true potential for σ is generated if both j and j' acquire vacuum expectation values. This is not all that restrictive as j and j' can be any operators on the $y = 0, \pi R$ branes, as the $U(1)$ gauge transformations vanish at $y = 0, \pi R$ and arbitrary 4D Lorentz-scalar operators can be coupled to $\Phi(x, 0)$ and $\Phi(x, \pi R)$.

3 Supersymmetric Wilson lines on S^1/Z_2

In the last Section we discussed the Wilson line on S^1/Z_2 and its interactions with brane fields in the 4D effective theory arising from 5D local interactions. In this Section we generalize the discussion to a 5D $N = 1$ supersymmetric theory compactified on S^1/Z_2 . An $N = 1$ 4D superfield description appropriate for such theories is developed in Refs.[13, 14] and we here follow the conventions of [14] and employ some of the results of Ref.[15].

3.1 The 4D $N = 1$ supersymmetric Wilson line

For 4D $N = 1$ supersymmetry, where a manifest superspace formulation is at hand, the supersymmetric Wilson loop has been discussed previously in [16, 17]. The natural

generalization of the Wilson line in the supersymmetric case is the solution to the parallel transport equation of the super gauge connection along a path in superspace:

$$\frac{dz^M}{ds} \nabla_M W(C) = 0. \quad (23)$$

Here z^M are superspace coordinates and ∇_M is the gauge covariant derivative in superspace. The formal solution to Eq.(23) is

$$W(C) \equiv \mathcal{P} \exp \left(- \int_{s_1}^{s_2} ds \dot{z}^A \mathcal{A}_A \right), \quad (24)$$

where \mathcal{A}_A is the super gauge connection and $z^A = e_M^A z^M$ are the flat superspace coordinates, where e_M^A is the supervielbein. In the conventions of Ref.[19], the supervielbein is

$$e_M^A = \begin{pmatrix} \delta_m^a & 0 & 0 \\ -i\sigma_{\mu\dot{\mu}}^a \bar{\theta}^\mu & \delta_\mu^\alpha & 0 \\ -i\theta^\rho \sigma_{\rho\dot{\nu}}^a \epsilon^{\dot{\nu}\dot{\mu}} & 0 & \delta_{\dot{\alpha}}^{\dot{\mu}} \end{pmatrix} \quad (25)$$

and the super gauge connection is given in terms of the fields U and V by

$$\begin{aligned} \mathcal{A}_\alpha &= -e^{2V} D_\alpha e^{-2V} \\ \mathcal{A}_{\dot{\alpha}} &= -e^{2U} \bar{D}_{\dot{\alpha}} e^{-2U} \\ \mathcal{A}_a &= \frac{i}{4} \bar{\sigma}_a^{\dot{\beta}\alpha} [-D_\alpha \mathcal{A}_{\dot{\beta}} - \bar{D}_{\dot{\beta}} \mathcal{A}_\alpha + \{\mathcal{A}_\alpha, \mathcal{A}_{\dot{\beta}}\}] \end{aligned} \quad (26)$$

and the hermiticity constraint $e^{2V} e^{-2U} = (e^{2V} e^{-2U})^\dagger$. Choosing $U \equiv 0$ just corresponds to a partial gauge fixing and defines the gauge connection and hence the Wilson line purely in terms of the vector superfield V . Under a super gauge transformation, the solution Eq.(24) transforms as

$$W(C) \rightarrow U(\Lambda(z_f)) W(C) U^{-1}(\Lambda(z_i)), \quad (27)$$

where now Λ is a chiral super field.

3.2 The S^1/Z_2 supersymmetric Wilson line

We are, however, interested in the Wilson line in 5D $N = 1$ supersymmetric theories on S^1/Z_2 in the case that the 4D $U(1)$ vector multiplet is projected out by the orbifold conditions. This changes the above construction in a manner we now explain. We follow the conventions of [14].

Upon dimensional reduction on a circle, 5D $N = 1$ supersymmetry leads to 4D $N = 2$ supersymmetry with an $SU(2)_R$ symmetry relating the two superfield transformations and the central charge p_5 . Only one of these supersymmetries is manifest in the 4D superfield formulation of 5D, $N = 1$ supersymmetry in terms of 4D, $N = 1$ superfields in Wess-Zumino gauge of Refs.[13, 14]. Fortunately, on S^1/Z_2 , the conservation of the fifth component of momentum, p_5 , is broken by the orbifold projection, breaking 4D $N = 2$

down to $N = 1$, and the surviving $N = 1$ can be chosen as the manifest supersymmetry in the 4D superfield formulation. Given these facts we can employ the result Eq.(24) if we are able to identify the correct replacement for V in terms of the original 5D field content.

The field content of a 5D $N = 1$ vector supermultiplet is a vector-field v^M , a real scalar Σ and an $SU(2)_R$ gaugino doublet λ_α^i , to which an $SU(2)_R$ triplet of real auxiliary fields is added for the off-shell multiplet. The off-shell action is given by

$$S = \int d^4x dy \frac{1}{g^2} \left\{ -\frac{1}{4}(F_{MN})^2 - \frac{1}{2}(\partial_M \Sigma)^2 - \frac{1}{2}\bar{\lambda}_i i\Gamma^M \partial_M \lambda^i + \frac{1}{2}(X^a)^2 \right\} \quad (28)$$

where F_{MN} is the field strength tensor.

As has been shown in [14], the supersymmetry transformations remaining after orbifolding are the supersymmetry transformations of the components of the vector superfield²

$$V = -\theta \sigma^m \bar{\theta} v_m + i\theta^2 \bar{\theta} \bar{\lambda}_L - i\bar{\theta}^2 \theta \lambda_L + \frac{1}{2}\theta^2 \bar{\theta}^2 (X^3 - \partial_5 \Sigma) \quad (29)$$

with the super gauge transformation

$$V \rightarrow V + \frac{1}{2}(\Lambda + \Lambda^\dagger) \quad (30)$$

and the chiral superfield

$$\Phi = (\Sigma + iA_5) + \sqrt{2}\theta(-i\sqrt{2}\lambda_R) + \theta^2(X^1 + iX^2), \quad (31)$$

if Φ has the inhomogeneous super gauge transformation

$$\Phi \rightarrow \Phi + \partial_5 \Lambda. \quad (32)$$

This Φ supergauge transformation is necessary to stay in Wess-Zumino gauge, see Ref.[14].

Now, Eq.(32) implies that for *odd* Φ , $\nabla_5 \equiv (\partial_5 + \Phi)$ ought to be interpreted as a covariant derivative, acting on a chiral superfield in the y -direction. For our case of *even* Φ , conservation of the Z_2 parity of the chiral superfield, of $U(1)$ charge q upon which the covariant derivative acts requires the modified definition

$$\nabla_5 \equiv (\partial_5 + q\epsilon(y)\Phi). \quad (33)$$

The covariant derivative acting on the real superfield V is however

$$\nabla_5 V \equiv \partial_5 V - \frac{1}{2}(\Phi + \Phi^\dagger) \quad (34)$$

independent of the parity choice. The superfield action

$$S = \int d^8z dy \frac{1}{g^2} \left[\frac{1}{4} (W^\alpha W_\alpha \delta^2(\bar{\theta}) + \text{h.c.}) + (\nabla_5 V)^2 \right] \quad (35)$$

²In order to get a non-vanishing Wilson line, we choose opposite Z_2 parities for gauge fields and supergauge transformations compared to [14], and thus we modify definitions accordingly as we go along.

reproduces the component field action Eq.(28) up to irrelevant surface terms with the supersymmetric field strength, $W_\alpha \equiv -\frac{1}{4}\bar{D}^2 D_\alpha V$.

We are now ready to identify the vector superfield gauge connection and, via Eq.(24), the supersymmetric Wilson line phase and its kinetic term. Since the full 5D (super)Poincare group is broken by the orbifold boundary conditions it is convenient to think of y as just a parameter of the theory, allowing us to work in terms of 4D superfields. We only need to consider the $N = 1$ supersymmetry remaining after the S^1/Z_2 compactification, so the extension of the 4D supersymmetric vielbein of Eq.(25) to our case simply involves, allowing x -space curved and tangent space indices, a and m , to take on the values 0, 1, 2, 3, 5. In fact for the paths of interest which stretch from one orbifold brane to the other with constant θ and $\bar{\theta}$, we can forget about the vielbein in the definition Eq.(24).

The parallel transport of the Wilson line in the y direction requires

$$\frac{dy}{ds} \nabla_5 W(C_5) = 0, \quad (36)$$

and from the gauge transformation of Φ in Eq.(32), it is clear that this is satisfied by

$$W(C_5) \equiv \exp \left(- \int_{s_1}^{s_2} ds \frac{dy}{ds} \Phi \right) \quad (37)$$

where the sign in the exponent is fixed by demanding the gauge transformation

$$W(C_5) \rightarrow e^{-\epsilon(y)\Lambda(y_f)} W(C) e^{\epsilon(y)\Lambda(y_i)}. \quad (38)$$

In other words we identify the supergauge connection in y -direction to be

$$\mathcal{A}_5 \equiv \Phi. \quad (39)$$

and the supersymmetric Wilson line

$$W(C_5) \equiv e^{-\Theta} \equiv \exp \left(- \int_0^{\pi R} dy \Phi \right). \quad (40)$$

The form of supersymmetric Wilson line in terms of component fields can be obtained from Eq.(40) by expanding out $W(C)$ in a power series of θ and $\bar{\theta}$. From the component expansion of Φ in Eq.(31) one can see that the θ and $\bar{\theta}$ -independent pure-phase part of this expression reproduces the expected Wilson line phase associated to A_5 . Analogous to the non-supersymmetric Wilson line, we interpret the supersymmetric Wilson line to be the exponential of a chiral (pseudo)Goldstone multiplet Θ .

The kinetic term of this multiplet arises from the original 5D bulk vector multiplet action. Explicitly, taking Φ to be in the ‘uniform’ gauge, i.e. setting

$$\Phi(x, y) = \Phi(x, 0), \quad (41)$$

independent of y , substituting this into Eq.(35) and integrating out y , we obtain the kinetic term for Θ :

$$\int d^8 z f^2 \Theta^\dagger \Theta + h.c. \equiv \int d^8 z \sigma^\dagger \sigma + h.c. \quad (42)$$

with $\sigma = f\Theta$ the canonically normalized 4D field where $f^2 = 2/\pi Rg^2$ similar to the non-supersymmetric case.

The most important feature of the supersymmetric Wilson-line, Eq.(40) is that it not only contains the real pseudoscalar field A_5 (or more precisely the pseudoscalar component of $W(C_5)$) but also the superpartners, the scalar Σ and the Weyl fermion λ_R as seen in the component expansion of Φ , Eq.(31). One possible application of this multiplet is as an ‘axion’ multiplet, so we will sometimes refer to the pseudoscalar, scalar, and Weyl fermion components of $W(C_5)$ as the axion, saxion, and axino, respectively, though in this paper we will not investigate such supersymmetric axion phenomenology in detail (see Refs.[12] and [18] for work along these lines in the non-supersymmetric context). As we discuss in Section 3.4 although the 4D field $W(C_5)$ originates from a 5D field it has the interesting property that it does not possess physical Kaluza-Klein mode excitations as one might naively expect.

3.3 4D Wilson line interactions from local 5D interactions

As we will show in this section, non-derivative interactions for the Θ supermultiplet arise in the effective 4D theory from charged 5D bulk hypermultiplets propagating between the branes in analogy to the non-supersymmetric Wilson line. For the general discussion of the hypermultiplet we again follow [13, 14]. We again specialize to $U(1)$ and modify the expressions according to our Z_2 parities.

The field content of a hypermultiplet is given by a complex scalar $SU(2)_R$ doublet H^i of opposite Z_2 parity and an $SU(2)_R$ singlet Dirac spinor Ψ which can be decomposed into two Weyl spinors $\Psi = (\psi, \bar{\psi}^c)^T$ of opposite Z_2 parity. A complex scalar $SU(2)_R$ doublet \tilde{F}_i of auxiliary fields of opposite Z_2 parity is added for the off-shell multiplet. The supersymmetric action is

$$S = \int d^4x dy \left(-(\partial_M H_i)^\dagger (\partial^M H^i) - \bar{\Psi} (i\Gamma^M \partial_M + \epsilon(y)m) \Psi \right). \quad (43)$$

Choosing the Z_2 parities as

$$\begin{array}{c|c|c|c|c|c} H^1 & \psi_L & F_1 & H^2 & \psi_R & F_2 \\ \hline + & + & + & - & - & - \end{array}, \quad (44)$$

the supersymmetry transformations of the component fields can be identified as the superfield transformations of the chiral superfields

$$\begin{aligned} H &= H^1 + \sqrt{2}\theta\psi_L + \theta^2(F_1 + \partial_5 H^2) \\ H_c &= H_2^\dagger + \sqrt{2}\theta\psi_R + \theta^2(-F_2^\dagger - \partial_5 H_1^\dagger), \end{aligned} \quad (45)$$

and the 5D action in terms of these superfields is

$$S = \int d^8z dy \left\{ H^\dagger H + H_c H_c^\dagger + (H_c(\partial_5 + \epsilon(y)m)H\delta(\bar{\theta}) + \text{h.c.}) \right\}. \quad (46)$$

The next step is to couple the hypermultiplet to the vector supermultiplet. Let H carry $U(1)$ charge q :

$$H \rightarrow e^{-q\epsilon(y)\Lambda} H, \quad H_c \rightarrow H_c e^{q\epsilon(y)\Lambda^\dagger} \quad (47)$$

where, as Λ is odd, we have to introduce $\epsilon(y)$. Then the action for the hypermultiplet becomes

$$S = \int d^8 z dy \left\{ H^\dagger e^{2q\epsilon(y)V} H + H_c e^{-2q\epsilon(y)V} H_c^\dagger + (H_c (\nabla_5 + \epsilon(y)m) H \delta(\bar{\theta}) + \text{h.c.}) \right\} \quad (48)$$

where

$$\nabla_5 H \equiv (\partial_5 + q\epsilon(y)\Phi) H. \quad (49)$$

defines the action of the covariant derivative. There is one subtlety in the above treatment:³ in gauging the Hypermultiplet, the definitions of the auxiliary fields have to be modified to

$$\begin{aligned} H &= H^1 + \sqrt{2}\theta\psi_L + \theta^2(F_1 + D_5 H^2 - \epsilon(y)\Sigma) \\ H_c &= H_2^\dagger + \sqrt{2}\theta\psi_R + \theta^2(-F^{2\dagger} - D_5 H_1^\dagger - \epsilon(y)H_1^\dagger\Sigma). \end{aligned} \quad (50)$$

From Eq.(48), we derive the superfield propagators in Appendix B. Note that, as indicated in the last chapter, we treat $\Phi(x, 0)$ as a background field and thus expect a dependence of the hypermultiplet propagator on $\Phi(x, 0)$. The result for the hypermultiplet propagator is:⁴

$$\Delta_H = \begin{pmatrix} (-\partial_5 + \epsilon(y)(m + \Phi))\frac{\bar{D}^2}{4\Box} & \frac{\bar{D}^2 D^2}{16} \\ \frac{D^2 \bar{D}^2}{16} & (\partial_5 + \epsilon(y)(m + \Phi^\dagger))\frac{D^2}{4\Box} \end{pmatrix} \begin{pmatrix} \mathcal{G}_1(y) & 0 \\ 0 & \mathcal{G}_2(y) \end{pmatrix} \delta^4(\theta - \theta') \quad (51)$$

where the 2×2 matrix indicates the $(H_c, H^\dagger) \times (H, H_c^\dagger)$ propagators. Imposing boundary conditions according to H being Z_2 even and hence H_c being odd, we calculate the full y dependent $\mathcal{G}_{1,2}$ in the appendix (see Eq.(72)). Note however, that the potential for the Wilson line from the coupling of the hypermultiplet to brane localized fields at $y = 0$ and $y = \pi R$ only depends on $\frac{1}{16}\bar{D}^2 D^2 \mathcal{G}_2(\pi R)$ corresponding to the $H^\dagger H$ propagator as H_c (and H_c^\dagger) are Z_2 odd. For the brane-to-brane case of interest, $y = \pi R$, and \mathcal{G}_2 reads

$$\mathcal{G}_2 = \frac{1}{\mathcal{N}'_2} e^{-\frac{q}{2}(\Phi - \Phi^\dagger)\pi R} \quad (52)$$

where

$$\mathcal{N}'_2 \equiv \frac{2 \sinh(p' \pi R)}{p'} (p^2 + |m + q\Phi|^2) \quad (53)$$

and

$$p'^2 \equiv p^2 + \left(m + \frac{q}{2}(\Phi + \Phi^\dagger) \right)^2. \quad (54)$$

³See [14] and [15] for additional discussion in the case of Z_2 gauge supermultiplet parities opposite to ours.

⁴As in the non-supersymmetric case, we fix one end of the propagator on the $y = 0$ brane.

In a similar fashion to the non-supersymmetric case this brane-to-brane propagator may be simplified in various limits, the most important of which is the case where the inter-brane separation is large in units of the bulk hypermultiplet mass. As for the non-supersymmetric Wilson line phase this results in the interactions of the ‘axion’ and its superpartners with brane-localized fields being exponentially suppressed by $\exp(-m\pi R)$. This is of importance for applications of this mechanism to model-building where it is desired that the masses and non-derivative interactions of the pseudo-Goldstone multiplet are suppressed relative to the fundamental scale M_5 , as well as being protected from possible global-symmetry-violating quantum gravitational effects.

3.4 Kaluza-Klein modes and the Wilson line

By using the y -dependent superfield formalism no expansion and resummation in Kaluza-Klein (KK) modes was needed to achieve our results. However, to make contact to the KK picture and clarify a few subtleties related to it, a few comments are in order. For the non-supersymmetric Wilson line discussed in Section 2 we calculated the Wilson line kinetic term and its potential in uniform gauge Eq.(12). As indicated, this is justified as any local gauge-field contribution can be gauged away. However the brane-to-brane Wilson line is gauge invariant and therefore not all information contained in the A_5 gauge field can be entirely gauged away on S_1/Z_2 . In particular the y -independent mode of A_5 is physical.⁵ Therefore, in terms of KK modes, the pseudo scalar field given by the Wilson line phase contains only a single physical zero mode while *all higher KK-modes correspond to unphysical gauge degrees of freedom*.

In the supersymmetric case the Wilson line multiplet contains the Wilson line phase as its imaginary scalar component. As only the zero mode is a physical mode of the scalar component (by the previous gauge transformation argument), this must be true for the whole supermultiplet too. However, as for the treatment in 4D superfields it was necessary to work in Wess-Zumino gauge Eq.(32). Therefore, all components of the super gauge transformation Λ apart from the one we use to fix uniform gauge for the A_5 component are fixed by Wess-Zumino gauge condition and cannot be used to gauge away higher KK-modes of Σ and λ_R . Instead in Wess-Zumino gauge the higher KK modes of Σ and λ_R are “eaten” by the KK-modes of the massive vector supermultiplet V , providing the bosonic longitudinal degree of freedom for the $A_\mu^{(n)}$ KK-modes and the necessary fermionic degrees of freedom for the gauginos λ_L to form massive KK Dirac fermions. Thus the Wilson line supermultiplet arising from a 5D vector supermultiplet on S^1/Z_2 provides a 4D chiral pseudo-Goldstone supermultiplet with a zero mode and *no higher KK-modes*.

⁵Another way to see this is that the gauge transformation needed to gauge away the constant field is $\Lambda = y$ which does not satisfy the boundary conditions on S_1/Z_2 .

4 Conclusions

In this paper we considered a $U(1)$ gauge theory on the five dimensional orbifold $\mathcal{M}_4 \times S^1/Z_2$, where A_5 has even Z_2 parity. We showed that this leads to a light pseudoscalar degree of freedom $W(x)$ in the effective 4D theory below the compactification scale arising from a gauge-invariant brane-to-brane Wilson line. As noted by Arkani-Hamed *et al.* in the non-supersymmetric S^1 case the 5D bulk gauge-invariance of the underlying theory together with the non-local nature of the Wilson line field leads to the protection of the 4D theory of the Wilson line field $W(x)$ from possible large global-symmetry violating quantum gravitational effects. We studied the S^1/Z_2 theory in detail, in particular developing the supersymmetric generalization of this construction, involving a pseudoscalar Goldstone field (the ‘axion’) and its scalar and fermion superpartners (‘saxion’ and ‘axino’). The global nature of $W(x)$ implies the absence of independent Kaluza-Klein excitations of its component fields. The non-derivative interactions of the Wilson line degree of freedom in the effective 4D theory arise from $U(1)$ charged 5D fields Φ propagating between the boundary branes. Because such effects are suppressed by $\exp(-\pi R m_\Phi)$ a small hierarchy between the inverse mass $1/m_\Phi$ of such bulk scalars and the size, πR , of the 5th dimension leads to an exponentially large suppression of the non-derivative couplings of the Wilson line. Thus the pseudo-Goldstone nature of the 4D Wilson line field is easy to maintain against quantum gravity effects, and quite naturally the pseudo-Goldstone field (and its superpartners) can have mass much smaller than the fundamental scales, M_5 , or $1/R$ of the theory. Given these noteworthy properties we believe that investigation of the model building uses of such Wilson line degrees of freedom arising from A_5 zero modes on $\mathcal{M}_4 \times S^1/Z_2$ and $\mathcal{M}_4 \times S^1/(Z_2 \times Z'_2)$ orbifolds is warranted.

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A Conventions and notation

We use the metric $ds^2 = \eta_{MN} dx^M dx^N = \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$, where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, $M, N = 0, 1, 2, 3, 5$ and $\mu, \nu = 0, 1, 2, 3$ and $0 \leq y \leq \pi R$ is the fundamental domain after S^1/Z_2 compactification. When dealing with fermions, we use the following representation of the γ matrices:

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \gamma_5 = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \quad (55)$$

where $\sigma_\mu = (1_2, \sigma_i)$, $\bar{\sigma}_\mu = (1_2, -\sigma_i)$ and σ_i are the σ matrices.

To fix our conventions for the 5D super gauge transformations, we choose

$$e^{\epsilon(y)2V} \rightarrow e^{\epsilon(y)\Lambda^\dagger} e^{\epsilon(y)2V} e^{\epsilon(y)\Lambda} \quad (56)$$

which agree with the conventions of [14] apart from the step functions which we introduce as we choose V , Λ to have odd Z_2 parity.

Via Eq.(26), Eq.(56) yields

$$\mathcal{A}_M \rightarrow e^{-\epsilon(y)\Lambda}(\mathcal{A}_M + \partial_M)e^{\epsilon(y)\Lambda}. \quad (57)$$

From this, the super gauge transformation for a Wilson line of a path with constant $\theta, \bar{\theta}$ is

$$W(C) \rightarrow e^{-\epsilon(y)\Lambda(x_f)}W(C)e^{\epsilon(y)\Lambda(x_i)}. \quad (58)$$

B Calculation of superfield hypermultiplet propagator

Expanding the Hypermultiplet super-action Eq.(48) leads to

$$S = \int d^8z dy \{ H^\dagger H + H_c H_c^\dagger + H_c(\partial_5 + \epsilon(y)m + q\epsilon(y)\Phi)H\delta(\bar{\theta}) + H^\dagger(-\partial_5 + \epsilon(y)m + q\epsilon(y)\Phi^\dagger)H_c^\dagger\delta(\theta) + O(V) \} \quad (59)$$

As for the non-supersymmetric theory, we consider the propagation of the hypermultiplet in the presence of a $\Phi(x, 0)$ background field. Thus we obtain the following free generating functional⁶

$$\begin{aligned} Z_{0,H}(J_H, J_H^\dagger, J_{H_c}, J_{H_c}^\dagger) &= \int \mathcal{D}H \mathcal{D}H^\dagger \mathcal{D}H_c \mathcal{D}H_c^\dagger \\ &\exp \left\{ i \int d^8z dy \left[(H_c, H^\dagger) M_H \begin{pmatrix} H \\ H_c^\dagger \end{pmatrix} \right. \right. \\ &\quad \left. \left. + (H_c, H^\dagger) \begin{pmatrix} -\frac{D^2}{4\Box} & 0 \\ 0 & -\frac{\bar{D}^2}{4\Box} \end{pmatrix} \begin{pmatrix} J_{H_c} \\ J_H^\dagger \end{pmatrix} \right. \right. \\ &\quad \left. \left. + (H, H_c^\dagger) \begin{pmatrix} -\frac{D^2}{4\Box} & 0 \\ 0 & -\frac{\bar{D}^2}{4\Box} \end{pmatrix} \begin{pmatrix} J_H \\ J_{H_c}^\dagger \end{pmatrix} \right] \right\} \\ &= \exp \left\{ -i \int d^8z dy (J_{H_c}, J_H^\dagger)_i \Delta_{5,H_{ij}}(z - z', y - y') \begin{pmatrix} J_H' \\ J_{H_c}^\dagger' \end{pmatrix} \right\} \end{aligned} \quad (60)$$

where M_H is

$$(M_H) = \begin{pmatrix} (\partial_5 + \epsilon(y)m + \epsilon(y)q\Phi)(-\frac{D^2}{4\Box}) & 1 \\ 1 & (-\partial_5 + \epsilon(y)m + \epsilon(y)q\Phi^\dagger)(-\frac{\bar{D}^2}{4\Box}) \end{pmatrix} \quad (61)$$

and the chiral propagator $(\Delta_{5,H})$ is defined by

$$\begin{pmatrix} \frac{\bar{D}^2}{4\Box} & 0 \\ 0 & \frac{D^2}{4\Box} \end{pmatrix} (M_H)(\Delta_{5,H}) \equiv - \begin{pmatrix} -\frac{\bar{D}^2}{4\Box} & 0 \\ 0 & -\frac{D^2}{4\Box} \end{pmatrix} \delta^8(z - z')\delta(y - y'). \quad (62)$$

⁶We denote $J(z, y) \equiv J$ and $J(z', y') \equiv J'$.

To solve for the propagator we invert (M_H) , using the method of supersymmetry projectors outlined in [19]. From Eq.(61):

$$M_H = \begin{pmatrix} -\square^{-\frac{1}{2}}bP_+ & 1 \\ 1 & -\square^{-\frac{1}{2}}cP_- \end{pmatrix} \quad (63)$$

where

$$b = \partial_5 + \epsilon(y)m + \epsilon(y)q\Phi \quad (64)$$

$$c = -\partial_5 + \epsilon(y)m + \epsilon(y)q\Phi^\dagger. \quad (65)$$

Inverting this via supersymmetry projector algebra and using it to solve Eq.(62) yields

$$\Delta_H = \begin{pmatrix} \frac{c}{\square-bc} \frac{\bar{D}^2}{4} & \frac{1}{\square-cb} \frac{\bar{D}^2 D^2}{16} \\ \frac{1}{\square-bc} \frac{D^2 \bar{D}^2}{16} & \frac{b}{\square-cb} \frac{D^2}{4} \end{pmatrix} \delta(z-z')\delta(y-y') \quad (66)$$

up to terms which vanish under a super-integral under which Δ_H is defined.

Fourier transformation in the non compactified directions and rewriting this leads to

$$\Delta_H = \begin{pmatrix} \frac{c}{\square-bc} \frac{\bar{D}^2}{4} & \frac{1}{\square-cb} \frac{\bar{D}^2 D^2}{16} \\ \frac{1}{\square-bc} \frac{D^2 \bar{D}^2}{16} & \frac{b}{\square-cb} \frac{D^2}{4} \end{pmatrix} \begin{pmatrix} \mathcal{G}_1 & 0 \\ 0 & \mathcal{G}_2 \end{pmatrix} \delta^4(\theta-\theta') \quad (67)$$

where $\mathcal{G}_{1,2}$ are solutions to

$$(-p^2 - bc)\mathcal{G}_1 = \delta(y-y') \quad (68)$$

$$(-p^2 - cb)\mathcal{G}_2 = \delta(y-y') \quad (69)$$

and subject to the appropriate boundary conditions.

As in the non-susy case, to calculate the propagator, we fix $y' = 0$. The equations read

$$\left[\partial_5^2 + \epsilon(y)q(\Phi - \Phi^\dagger)\partial_5 - 2\delta(y)(m + q\Phi^\dagger) - p^2 - (m + q\Phi)(m + q\Phi^\dagger) \right] \mathcal{G}_1 = \delta(y) \quad (70)$$

$$\left[\partial_5^2 + \epsilon(y)q(\Phi - \Phi^\dagger)\partial_5 + 2\delta(y)(m + q\Phi) - p^2 - (m + q\Phi)(m + q\Phi^\dagger) \right] \mathcal{G}_2 = \delta(y) \quad (71)$$

As H is even and H_c is odd, we have to chose even and odd boundary conditions for \mathcal{G}_2 and \mathcal{G}_1 respectively. With these boundary conditions, the solutions to Eq.(70) are

$$\mathcal{G}_1 = e^{-\frac{q}{2}(\Phi - \Phi^\dagger)|y|} \frac{1}{\mathcal{N}_1} \sinh(p'(|y| - \pi R)) \quad (72)$$

$$\mathcal{G}_2 = e^{-\frac{q}{2}(\Phi - \Phi^\dagger)|y|} \frac{1}{\mathcal{N}_2} \left(p' \cosh(p'(|y| - \pi R)) + \frac{q}{2}(\Phi - \Phi^\dagger) \sinh(p'(|y| - \pi R)) \right) \quad (73)$$

where

$$p'^2 \equiv p^2 + \left(m + \frac{q}{2}(\Phi + \Phi^\dagger) \right)^2 \quad (74)$$

and the normalization factors are

$$\mathcal{N}_1 \equiv q(\Phi - \Phi^\dagger) \sinh(p'\pi R) - 2p' \cosh(p'\pi R) \quad (75)$$

$$\mathcal{N}_2 \equiv 2 \sinh(p'\pi R) (p^2 + |m + q\Phi|^2). \quad (76)$$

Evaluating \mathcal{G}_2 at $y = \pi R$ and absorbing p' into the normalization then leads to the result given in Eq.(52).

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